Dynamic Periodic Table of the 2×2 Games: User's Reference and Manual

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Abstract

Use this manual both as a tutorial guide on how to use the *Dynamic Periodic Table of the* 2×2 *Games* software and as a reference for interpreting the displays. The logic behind the system for organizing the ordinal 2×2 games is briefly explained but a full elaboration can be found in Robinson and Goforth[2].

Contents

1 Introduction

The Dynamic Periodic Table of the 2×2 Games is an interactive program for exploring the relationships among the ordinal 2×2 games, the classic set of games at the foundation of game theory. The software is based on the 2005 book, *Topology of the* 2×2 *Games: a new periodic table*, by the same authors[2].

The periodic table displays the set of games based on an organization that locates games close to others that are similar according to a formal definition of neighbourhood. An indexing system based on this organization uniquely identifies each of the 144 games. There are interactive controls to move games around to highlight different relationships.

The program contains a lot of information about individual games. There are also controls to specify which information is displayed on the periodic table.

1.1 Quick Start

To launch the program, click the button "Initialize display". The periodic table is best viewed on a screen of at least 22".

1.2 Access to the Software

The Dynamic Periodic Table of the 2×2 Games is available as an on-line applet to run directly at http://www.cs.laurentian.ca/dgoforth/home.html and also as a Netlogo source code application that can be downloaded to be run locally. The program was developed in Netlogo 4.1. Netlogo is available at no charge from Uri Wilensky, Northwestern University, at http://ccl.northwestern.edu/netlogo/ for various platforms.

1.3 Organization of the Manual

In section 2, the periodic table is introduced as an organizing structure for the strictly ordinal 2×2 games. This section assumes basic understanding of game thory and 2×2 games. For a more complete introduction, see the book[2].

Section 3 describes the layout of the screen and introduces the basic information and controls. There are two categories of controls. One set, with the title "Arrange periodic table" for rearranging the games in the display, is described in detail in Section 4. The other set, entitled "Game attributes and information", determines what information about the games is shown in the display. The description is in Section 5.

2 Periodic Table

The periodic table [2] is a preference-based organization of the 2×2 ordinal games that locates similar games close to each other. If the organizational principle is valid, we would expect global patterns to emerge when data gathered across many games are displayed on the table and this is indeed the case.

2.1 The structural logic of the periodic table

A player of a strict ordinal 2×2 game sees one of only six possible patterns of her own payoffs. These are presented and indexed for the row player in Figure 1 that shows the payoff matrices for the row player only. With each pattern, she might face one of six possible patterns of her opponent's payoffs. For a specific pair of payoff patterns, there are four ways to combine them to create distinct games. In all there are $6 \times 6 \times 4 = 144$

	L	R	L	R	L	R	L	R	L	R	L	R
U	1	3	2	3	3	2	3	1	2	1	1	2
D	2	4	1	4	1	4	2	4	3	4	3	4
Index:	1		2		3		4		5		6	

Figure 1: Patterns of Row's Payoffs

strict ordinal games. This is the set of games portrayed on the Dynamic Periodic Table of the 2×2 Games.

Similarity of games is defined structurally by comparing payoff matrices. For any game, those most similar to it are the games that differ only in that a pair of ordinally consecutive payoffs (1 and 2, 2 and 3, or 3 and 4) for one player are in exchanged positions in the matrix. Hence, every game has six adjacent neighbours and we can represent the ordinal 2×2 games as a graph of 144 nodes with each connected by 6 edges to other game nodes[1].

This graph of 144 nodes and 432 edges cannot be mapped onto a plane so displaying it requires some decisions about which adjacencies to highlight. We do so based on the presumption that swapping high-valued payoffs 3 and 4 will alter the character of a game more than swapping low-valued payoffs 1 and 2 or 2 and 3 that are less likely to appear in solution outcomes. By concentrating on the four low-valued swaps, the periodic table will show games near those to which they are generally most similar.

The resulting graph with nodes of degree four turns out be four disconnected toruses that are easily represented as 6×6 "layers" of games as in Figure 2. Based on the initial observation that there are six patterns for each player, we can identify the rows of each layer with particular row player patterns and the columns with column patterns. Thus the games can be indexed according to array position. If the four layers are piled up so that row patterns and column patterns coincide, then the four games with the same row and column index form a 'stack'. In every stack are the four distinct games constructed from the same row and column payoff patterns.

To each game is assigned a three digit index designating its *layer*, *row* and *column*. For example, game 214 in Figure 2 is on the second layer in row one at column four. Figure 3 shows the stack of four games, including game 214, that share the same row and column

[1] We call the transformation of a game into one of its neighbours a 'swap'. The six swap operations are named C12, C23, C34, and R12, R23, R34. So that the adjacency based on swapping a pair of payoffs is unconstrained, we do not reduce the set to 78 games by the common assumption of equivalence under the exchange of player roles.

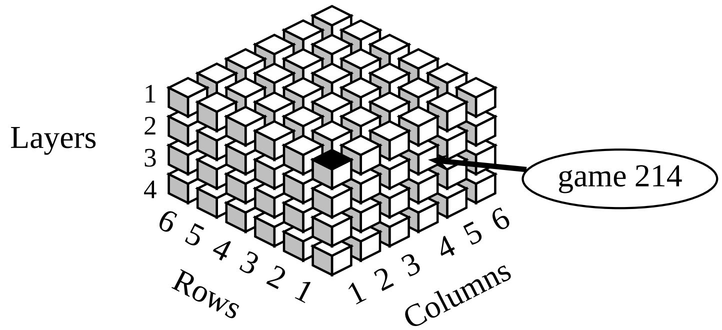

Figure 2: Arrangement of the 2×2 games by indices

payoff patterns.

As well as payoff matrices, Figure 3 shows the more visual and convenient representation in payoff space, called an *order graph*, that is employed in one of the displays of the periodic table. See Subsection 5.6. Row payoffs are plotted on the horizontal axis with column payoffs vertical. Each point represents an outcome and the edges represent inducement correspondences [1] corresponding to Row's choices (double line for a column) and Column's (single line for a row) on the payoff matrix that can be "induced" by the opponent. In Figure 3, the open circles in the order graph represent Nash equilibria. This is only one of the features that can be displayed on the dynamic version. Again, see Subsection 5.6.

In the periodic table, the four layers are arranged in a two by two array creating a 12×12 display of games. Clearly, the configuration is still underconstrained as there is choice in where to cut and project the toruses to form layers and there is choice in how to place the four layers. In the static printed version of the table, we have made specific decisions but in the dynamic version, the user can move, roll and reflect the layers to feature different adjacencies, including the swaps of 3 and 4 values that connect games on different layers. Manipulation of the layout of the display of the periodic table is described in Section 3.

2.2 Emergent properties of the periodic table

Although the adjacency of games on the periodic table is defined in strictly structural terms, there are many basic properties, including some used in defining taxonomies, that emerge as clustered regions of the table.

Game 114

	L	R
U	1,4	3,1
D	2,2	4,3

Game 214

	L	R
U	1,2	3,3
D	2,4	4,1

Game 314

	L	R
U	1,3	3,2
D	2,1	4,4

Game 414

	L	R
U	1,1	3,4
D	2,3	4,2

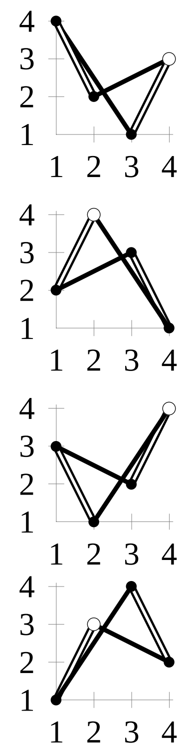

Figure 3: A stack: four games with the same payoff patterns for each player. The double line corresponds to a column in the payoff matrix; the open circle indicates a Nash equilibrium outcome.

2.2.1 Player patterns

The six patterns for one player form a closed cycle on a layer under consecutive swaps of 1 and 2 then 2 and 3, either C12 and C23 or R12 and R23. The three patterns that contain a dominant strategy are adjacent (indices 1, 5 and 6 in Figure 1) as are the three that do not have a dominant strategy (2, 3 and 4). The pattern labeled 1 is the weakest of the dominant strategy patterns, the only one with a dominant payoff of 2. Player payoff patterns, as defined in Figure 1 can be explored on the dynamic table - see Subsection 5.3.

2.2.2 Symmetry and Player Exchange

In a symmetric game, both players have the same payoff patterns, but only two of the four relative orientations produce a symmetric game. The twelve symmetric games are on layers 1 and 3, where the row and column indices are equal. Games with the same row and

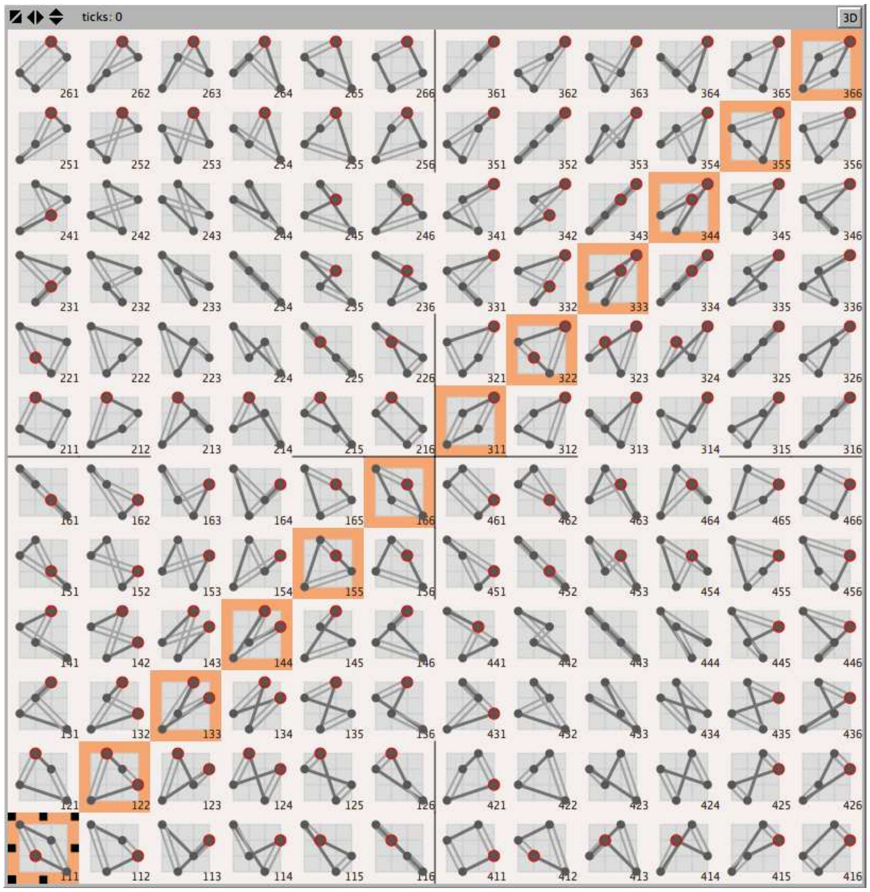

Figure 4: Twelve symmetric games on the diagonals of layers 1 and 3

column index on layers 2 and 4 share the same pattern but are not symmetric. To highlight and examine the symmetric properties, see Subsection 5.3.

The games that are equivalent under an exchange of player roles can be paired by reflecting across the main positively sloped diagonal on the full periodic table in its standard configuration. See Subsection 1.1. For example, game 214 is the “reflection” of game 441 and 326 reflects 362. Considering only the 66 games one side of the diagonal together with the 12 symmetric games produces the count of 78 games defined by Rapoport and Guyer in [3]. See Subsection 2.2.5.

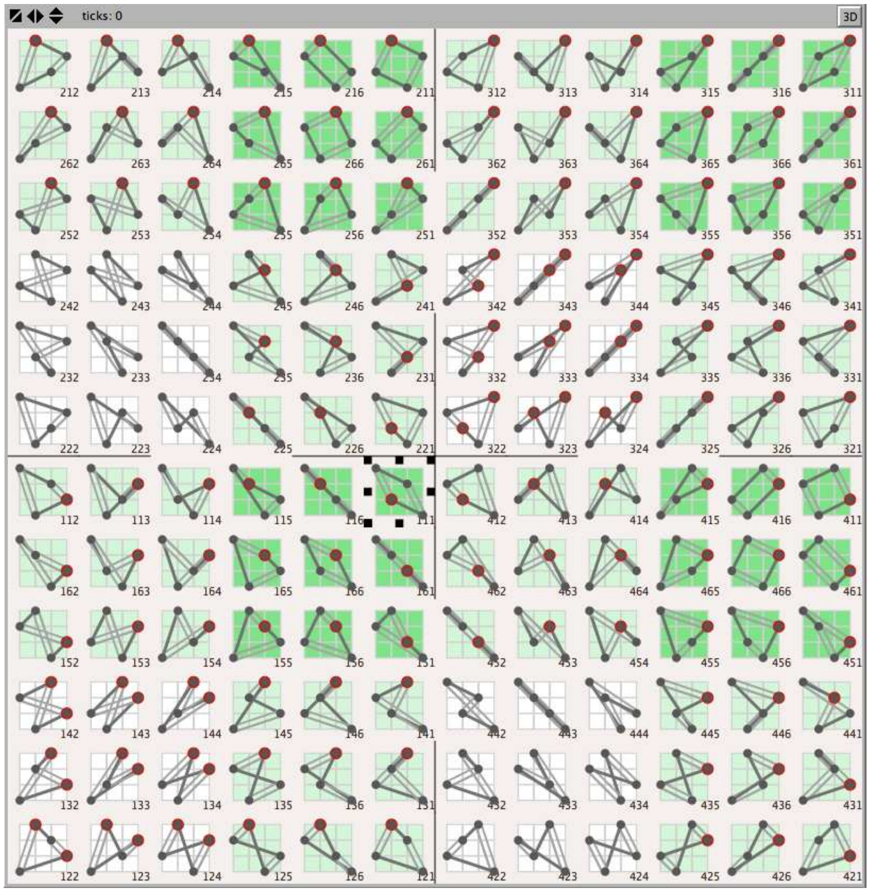

Figure 5: Layer Quadrants highlighting Dominance

2.2.3 Combinations of patterns - dominant strategies

Each layer can be arrayed with the row player's dominant strategy patterns in the top three rows and the column player's dominant strategy patterns in the rightmost columns. Where these patterns meet in the top right corner of each layer are nine dominant strategy games. The nine in the top left are dominance-solvable because the row player has a dominant strategy as are the nine in the bottom right because the column player has a dominant strategy pattern. In the bottom left are the games with no dominant strategy. These descriptions apply to all layers. Figure 5 shows the periodic table in this arrangement (See Subsection 4.4) with the dominant strategies highlighted. See Subsection 5.4.

2.2.4 Equilibria

All dominant strategy and dominance-solvable games, the games shaded in Figure 5, have a single Nash equilibrium. All the games with zero or two equilibria are composed of player patterns without a dominant strategy. These are the games in the bottom left quadrant of each layer in the Figure, where both players have patterns 2, 3 or 4. The games on layers 2 and 4 have no equilibria. Those on layers 1 and 3 have 2 equilibria but they are further distinguished. On layer three, all the games have one equilibrium that Pareto-dominates the other. These are the so-called coordination games including Stag Hunt (game 322). On layer one are the Battle-of-the-Sexes games, including Chicken (game 122), with two undominated equilibria.

2.2.5 Rapoport and Guyer's Taxonomy

In Figure 6, the games are coloured and indexed according to Rapoport and Guyer's taxonomy[3]. Reflected asymmetric games have the same index. Except for the green shaded games (67, 68, 69) "two equilibria with non-equilibrium solution", all of the taxonomic categories are contiguous in the periodic table. Some of the adjacencies are 3-4 swaps not shown in the configuration of Figure 6.

The "no equilibrium" category (blue) on layers 2 or 4, is exactly the set identified as having no dominant strategy for either player in Figure 5.

Also, the "no conflict" category (white in Figure 6) consisting of all games with an outcome of (4,4) is exactly layer three of the periodic table.

3 Screen Organization

The interface is divided vertically into three areas. At the centre is the graphic display of the periodic table, showing information about the 144 games. To the right is a text window that summarizes the graphic display and contains more information about one of the games. To the left of the graphic display is a command centre with controls allowing the user to manipulate the graphic display and the information appearing in the text window.

3.1 Graphic Display of the Periodic Table

The graphic display of the periodic table is composed of a twelve by twelve grid of cells. Each cell shows some information about one of the games. The games can be moved about but some features of the display are fixed:

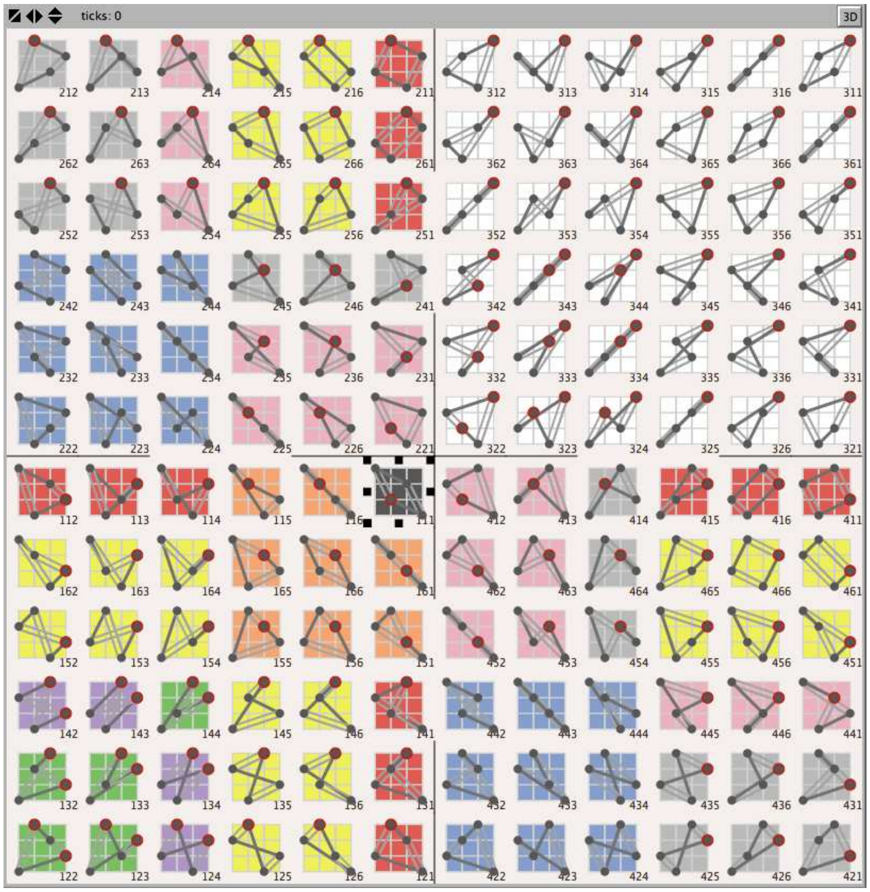

Figure 6: Rapoport and Guyer Taxonomy on the periodic table

- The games on one layer of the periodic table are alway shown together in one six by six quadrant of the table. Short black lines indicate the boundaries separating quadrants. Entire layers can be exchanged between quadrants.

- Within a layer, adjacent games (above, below, left, right) are always neighbours in terms of swap operations, C12, C23, R12 and R23. Games at the right edge of a quadrant are adjacent to games at the left edge in the same row and games at the top of a quadrant are adjacent to games at the bottom edge in the same column. Recall that neighbours under C34 and R34 swaps are on different layers and rarely adjacent on the periodic table. Subject to these constraints, games can be moved around within a quadrant.

The appearance of the periodic table can vary depending on what information is being displayed or hidden. However, any item of information will be shown for all games.

3.1.1 Anatomy of one Cell

There are several features of a cell that convey information about a game.

1. “Subset”: border colour that appears as a “frame” around the cell. A range of colours is used to group games into subsets according to the periodic table structure. For example, in Figure 4, symmetry is shown with two colours, one to frame symmetric games and a second to frame asymmetric games.

2. “Index”: integer at bottom right of the frame. A number may appear to identify the index of a game. In Figure 4, the index as defined in [2] is displayed on each game cell.

3. “Property”: background colour inside the frame. This colour identifies properties of the game that are not explicitly functions of the periodic table structure. In Figure 5 for example, three colours are used to identify games in which both players (green), one player (pastel green) or none (white) have a dominant strategy.

4. “Data”: many graphic features, including diagrams and bar charts can be superimposed on the background to display other data about a game. In all the figures above, the order graph is displayed.

5. Current Game: one cell on the display is highlighted with sequence of black dots around the frame. In the above figures, game 111 is highlighted. This is designated the “current game”, the game whose complete information is printed in the text window to the right of the graphic display. See Subsection 3.2.

The information on the graphic display is presented in colours and symbols. While the meaning of many of the displays will be obvious, interpretation of others depends on a legend. Brief descriptions are included in the text window at the right and complete explanations of the displays are found in Section 5 of this reference manual with the controls for showing or hiding them.

3.2 Text Window and the Current Game

At the right of the screen is a text window with two kinds of information. At the top, there is a brief description of the content of the current graphic display.

Below, there is comprehensive information about the current game (See Subsection 3.1.1), identified by its index in the periodic table. Following the index are any common names by which it is known (e.g., “Chicken”) and indices in other classification systems.

The maximum total outcome is listed, then the payoff bi-matrix is shown followed by solution outcomes from various models in the literature.

The data in the text window is always updated to match the current state of the graphic display and the current game.

3.3 Interactive Controls

At the left of the screen are the controls for the graphic display and the text window. The instructions for using the controls are found in Sections 4 and 5.

4 Interactive Controls: Arrange Periodic Table

The Periodic Table consists of 144 games with 432 ‘swap’ links connecting games to their closest neighbours. This graph structure cannot be displayed in its entirety on a flat surface. The dynamic version of the Periodic Table allows reconfiguration of the games to represent various features of the graph. In this section, the controls for manipulating the configuration of the table are explained.

4.1 Basic concepts of the display

Each layer of the table is a torus. To show it in a ‘flat’ quadrant of the graphic display, the torus must be cut in two orthogonal directions and laid out in a six by six array. These cuts can be made between any rows and columns. In the display, the four quadrants are always cut the same way. If one layer is cut between rows 3 and 4, all the layers are cut between rows 3 and 4. In Subsection 4.4, the controls for changing the cutlines, or “rolling the torus”, are described.

The four layers are shown in the four quadrants of the graphic display. Where the edges of quadrants meet along the vertical and horizontal median lines, there may or may not be a neighbour relationship between games in different quadrants that are adjacent in the display. That will depend on which layers are adjacent and how they are cut. In Subsection 4.5, controls are described for moving layers between quadrants. See also Subsection 5.7 on how to highlight neighbour links across quadrant boundaries.

Sometimes visual patterns of common features are more evident as reflections than as translations. The games of layers can reordered vertically or horizontally to create a reflected display. The controls to reflect layers are described in Subsection 4.6

4.2 Initialize Display - button

When the program is initialized (Click "Initialize idsplay"), the periodic table is displayed in *standard configuration* with the four layers located clockwise from the bottom left:

2	3
1	4

On each layer, the rows are arranged in ascending order bottom to top while the columns are ordered left to right. Thus games 111, 211, 311, 411 are in the bottom left corners of the quadrants.

261	...	266	361	...	366
..		..	..		..
211	...	216	311	...	316
161	...	166	461	...	466
..		..	..		..
111	...	116	411	...	416

In the standard configuration, the twelve symmetric games occupy the positive-sloped diagonal across layers 1 and 3. These games are numbered 111, 122, up to 166 then 311, 322 up to 366. The asymmetric games form two complementary sets above and below the symmetric diagonal. If the display is *folded* along the diagonal (along the line of symmetric games in Figure 4), every asymmmetric game maps to its complement, the same game with the player roles exchanged. For example, game 261 maps to game 416. Layer two maps onto layer four; both layers one and three maps onto themselves.

The display can be returned to standard configuration by clicking the "Initialize display" button again. The features selected for display will not change except that the current game will be reset to game 111.

4.3 Animated? - switch

When any of the controls for rearranging the period table is clicked, some or all of the games move to a new location on the display. To help visualize the transformation, the rearrangement can be animated. Symbols representing the games are seen to travel from the old location to the new. If the switch "Animated?", under the "Initialize display" button, is turned on, the animation will appear; if it is turned off, the old configuration is simply and quickly replaced by the new one.

For more control, the speed of the animation can be changed. Open the source code by clicking the "Procedures" tab, then search, using the "Find..." key at top left, for the statement “set steps” and change the value of the assignment. The standard value is 20. For a slower transition, change 20 to a larger number. Return o the display format by clicking the “Interface” tab.

To slow down one transition temporarily, pull the speed slider to the left from “normal speed” to “slower”.

4.4 Roll layers as torus - buttons

To understand the effect of these commands, be sure the “Animated?” control is set to “On” while you are first exploring them.

To change the cutlines on the toruses of each layer, use the four controls with the title “Roll layers as torus”. The effect of changing the vertical cut line is to move a column of six games from one edge to the other. Move the leftmost column to the right edge by clicking the “Left” button. Notice in the animation that the remainder of the columns are moving one position to the left at the same time. In fact the buttons are labeled to show the direction that most of the games will move. Also notice that all four layers rotate concurrently.

If some of the layers have been reflected (See Subsection 4.6), the rotations may appear to move in the *opposite* direction. This is necessary to maintain the relationship among the quadrants. Any command will always apply directly to the layer that occupies the bottom left quadrant. Any opposite effect will occur in some other quadrant(s).

The four rotation commands are:

- ”Left”: slide columns left; move leftmost column to right side.
- ”Right”: slide columns right; move rightmost column to left side.
- ”Up”: slide rows up; move top row to bottom.
- ”Down”: slide rows down; move bottom row to top.

There are shortcut keys defined on the buttons for each of these operations.

4.5 Exchange layers - buttons

To understand the effect of these commands, be sure the “Animated?” control is set to “On”.

The arrangement of the layers in the quadrants can be changed from the standard configuration by using the six keys with the title “Exchange layers”. Clicking any of these keys causes the layers currently occupying two quadrants to change places.

The six exchange commands are:

- ”Lt ‖”: exchange layers in top and bottom quadrants on left side.
- ”‖ Rt”: exchange layers in top and bottom quadrants on right side.
- ”Top ==”: exchange layers in top left and right quadrants.
- ”Bottom ==”: exchange layers in bottom left and right quadrants.
- ”\\”: exchange layers in top left and bottom right quadrants.
- ”//”: exchange layers in top right and bottom left quadrants.

If the layers that are exchanged were reflected relative to eachother before the exchange (See Subsection 4.6), then reflections will occur after the exchange.

4.6 Reflect half of table - buttons

To understand the effect of these commands, be sure the “Animated?” control is set to “On”.

The two buttons with the title “Reflect half of table” control the reflection operations. In the standard configuration, the layers are displayed with the rows indexed from bottom to top and the columns from left to right. The reflection commands allow the orientation of layers to be changed so the left side of the periodic table appears to “reflect” the right side and the top half reflects the bottom half.

The two reflection commands are:

- “TOP /\\/”: reverse the order of rows in the top two quadrants. The rows are now indexed from top to bottom. Clicking the button again undoes the reflection, restoring the bottom to top row indexing.
- “>< RIGHT”: reverse the order of columns in the two right quadrants. The columns are now indexed from right to left. Clicking the button again undoes the reflection, restoring the left-to-right column indexing.

5 Interactive Controls: Game attributes and information

In Subsection 3.1.1, the structure of one game cell in the graphic display is explained. The controls for selecting information to show in the cells are described in this section. These controls are grouped under the title “Game attributes and information”. The general approach is to choose one or more display items from the green controls, then click the

"Update display" button beside the title. The selected information will be shown but the arrangement of the cells in the display will not be changed. To show the new information while restarting in the standard configuration, click the "Initialize display" button.

5.1 Current Game - input

Complete information about one game, called the *Current Game*, is displayed in the text window. When the display is initialed, game 111, Prisoner's Dilemma, is the current game. To select a different current game, enter the Robinson and Goforth index of the game in the Input text box labeled "CurrentGame" beside the subtitle "1. Select game".

A game index is comprised of three digits. The first identifies the layer, 1 to 4; the second identifies the row, 1 to 6 and the third identifies the column, also 1 to 6. For example, 352 selects a game on layer three, row five, column two. If an incorrect game index is entered, the current game is not changed.

The current game is used in defining the relative subsets. See Subsection 5.3.

5.2 Game Index - chooser

Under the title "2. Game Attributes and Information", the first chooser is "Index". To determine which indexing system is used on the graphic display, make a selection from this chooser. There are four choices:

- "None": Show no index on the game cell.
- "Robinson and Goforth": Show the periodic table index of each game in the game cell. See Section 2 for an explanation of the indexing system.
- "Rapoport and Guyer": Show the index of each game based on the tabulation from Rapoport and Guyer[3]. Only 78 games were identified in this system, so asymmetric games with the player roles reversed have the same Rapoport and Guyer number. For example games 216 and 461 on the periodic table are both identified with the Rapoport and Guyer number 17.
- "Brams": Show the index of each game based on the tabulation from Brams[4]. Brams considered only 57 games of interest, ignoring games that occupy the third layer in the periodic table. For example, game 325 in the periodic table has no Brams index. Also, asymmetric games with the player roles reversed have the same Brams number. For example games 216 and 461 on the periodic table are both identified with the Brams number 5.

Note that all indices for the current game are always displayed in the text window.

5.3 Subset - chooser

Under the title "2. Game Attributes and Information", the second chooser is "Subset". Selecting a subset determines the colouring of the frames around the game cells. Subsets of the games that share a common feature will share a frame colour.

All the features in this chooser are defined in terms of the topology of the periodic table and the "swap" transformations. (See Section 2.) The first group of subsets display games with symmetric features. The second group, below the dashed line in the chooser list, define subsets connected to the current game, hence the set that is features may change if a different current game is specified. (See Subsection 5.1.)

The symmetry-based subset selections are:

- "None": Show no subsets. All games cells are framed in a common neutral colour.
- "Symmetric Games": Show the twelve symmetric games framed in a distinct colour. This subset is shown in Figure 4.
- "Same Pattern": Show the twenty-four games in which both players face the same pattern. Twelve of these are symmetric games and twelve are asymmetric and occur in reflected pairs. (See [2], p. 70.)
- "Quasi-symmetry": Show the twenty-four games with quasi-symmetry: they are symmetric under an exchange of payoffs between players at every outcome.
- "Symm & Quasi-symm": Show the forty-eight games that either have the same patterns (as above) or are quasi-symmetric (as above).

The selections for subsets relative to the current game are:

- "Stack": Show the games that occupy the same stack as the current game. A stack consists of four games, one on each layer, that have the same payoff pattern for the row player and the same payoff pattern for the column player. For example, games 134, 234, 334, and 434 form a stack. (See [2], p. 50.)
- "Neighbours": Show current game and the six games that are adjacent to it by a single swap transformation. The list of neighbours of the current game is also displayed in the text window. The concpt of "neighbours" is explained in Subsection 2.1.
- "Slice / Row Pattern": Show the twenty-four games that share the same payoff pattern for the row player of the current game. (See [2], p. 48.)

- “Slice / Column Pattern”: Show the twenty-four games that share the same payoff pattern for the column player of the current game. (See [2], p. 48.)
- “Tile R12 C12”: Show the four games accessible from the current game by R12 and C12 swaps. (See [2], p. 46.)
- “Pipe or Hotspot”: Show the games accessible from the current game by R12, R34, C12 and C34 swaps. If the set consists of eight games on two layers, it is a *hotspot*; if there are sixteen games on four layers, it is a *pipe*. (See [2], p. 97, 109.)

5.4 Property - chooser

Under the title “2. Game Attributes and Information”, the third chooser is “Property”. Selecting a property determines the colouring of the background of the game cells, inside the frame. Games that share a common property will share a backgound colour.

Properties are not based on the structure of the periodic table; they are features that have been identified by study of their emergent features of play. The selections for properties are:

- “None”: Show no properties. All cells have the same background colour.
- “Dominant Strategies”: Show games in which no player (white background, 36 games), one player (light colour, 72 games) or both players (dark colour, 36 games) have a dominant strategy. This property is shown in Figure 5.
- “Dominant Preferences”: Show games in which no player (white background, 36 games), one player (light colour, 72 games) or both players (dark colour, 36 games) have a dominant preference. This property (identified by Perlo-Freeman [5]) defines a pattern in which a player always prefers that one of the opponent’s strategies be played, no matter what strategy she is playing herself.
- “Nash Equilibria”: Show games with zero Nash equilibria (blue background, 18 games), one Nash equilibrium (white background, 108 games) or two Nash equilibria (pink background, 18 games).
- “Conflict/Common Interest”: Show the pure conflict games (blue background, 14 games), the pure common interest games (pink background, 14 games) and the *type* games (violet, 12 games) in which one player’s choices are purely common interest and the other player’s choices are in conflict. (See [2], p. 124.)

- “Max Total Payoff”: Show the total (ordinal) payoff possible in each game, either 5 (darkest, 6 fixed-sum games), 6 (dark, 42 games), 7 (light, 60 games) or 8 (white, 36 games on layer 3).

- “Rapoport Categories”: Show the categories identified by Rapoport and Guyer[3]. Theses categories are illustrated in Figure 6.

  - Dark Gray: Strongly stable deficient. 1 game.
  - White: No conflict. 36 games.
  - Pink: Force vulnerable. 18 games.
  - Red: Threat vulnerable. 12 games.
  - Orange: Strongly stable. 8 games.
  - Yellow: Stable. 24 games.
  - Green: 2 equilibria with non-equilibrium. 5 games.
  - Blue: Without equilibria. 18 games.
  - Violet: 2 equilibria with equilibrium. 4 games.
  - Light Gray: Unstable. 18 games.

  The category of the current game is named in the text window.

5.5 Data - chooser

Under the title “2. Game Attributes and Information”, the fourth and final chooser is “Data”. In this category are several displays that portray data about individual games. One of the selections, “Order Graphs” is an alternative representation of the data in a payoff matrix and is described in detail in the following subsection, 5.6. Many of the displays use a bar graph format that can easily be adapted to other data sets. See Section 6

Below the dashed line are several data displays based on the work of John Miller. These data are included as an example of how the periodic table can be used to visualize and study data from experiments on the set of ordinal 2×2 games.

- “None”: Show no data on the game cells.

- “Order Graph”: Show the order graph of each game against the ‘Property’ background. For details of the order graph display, see the next subsection, 5.6.

- "Solutions": Show the bar graph that compares solutions of the games according to various models of play. Five different solutions are graphed: Nash bargaining solution, Kalai-Smorodinski solution, Maximin, and the Nash equilibria, if any, as well as Miller's solutions. The bar graph contains from 8 to 12 bars depending on the number of Nash equilibria in the game. A pair of bars in light and dark shades of a colour represent the payoffs for the row and column players respectively. From left to right in the bar chart, here are the colours.
  - Gray: Nash bargaining solutions
  - Blue: Kalai-Smorodinski solutions
  - Orange: Miller average automata solutions
  - Green: Maximin solutions
  - Red: Nash equilibria - may be absent or appear twice
- "Miller Row Complexity": Show the complexity measure for the *row* player as a bar. The values are normalized in a distribution with mean value zero. Black values indicate complex behaviour and red values are relatively simple. The highest bars indicate the most extreme values, complex or simple.
- "Miller Column Complexity": Show the complexity measure for the *column* player as a bar. The values are normalized in a distribution with mean value zero. Black values indicate complex behaviour and red values are relatively simple. The highest bars indicate the most extreme values, complex or simple.
- "Miller Complexities": Show the complexity measures (positive and negative in a normalized distribution) for the two simulated players as two bars. The left bar measures the row player and the right measures the column player. The positive (black) values indicate complex behaviour and red values are relatively simple.
- "Miller Complexity Diff": Show the difference in complexity between the two players in the game as a bar. Blue indicates that the row player is more complex and yellow indicate column is. A larger bar is associated with a greater difference in complexity between the two players.
- "Complexities / Payoffs": Show four bars displaying both complexity of players and success at achieving long term average payoffs. The leftmost and rightmost bars, in black, indicate the complexity of the row and column players respectively. Only complex strategies are shown so all simple strategies are blank. The two central bars measure payoff success. Success is calculated as long term average payoff minus

the Kalai-Smorodinski solution payoff for the game. If the measure is negative (i.e., the player has fallen short of the K-S potential), the bar is orange. Green indicates the player has exceeded the K-S solution payoff.

5.6 Order Graph - switches

The order graph is defined in Subsection 2.1. It can be selected as the "Data" display item. See previous Subsection. On the order graph, it is possible to plot the game outcomes based on various solution concepts[2]. Below the "Data" chooser are a set of switches for specifying which outcomes should be displayed when order graphs are drawn on the periodic table.

- "Nash-bargaining?": Black +
- "Maximin?": Green dot
- "Miller?": Green $\times$ - Miller's average automata solutions
- "Nash equilibrium?": Red circle - Nash equilibria may be absent or appear twice
- "Kalai-Smorodinski?": Blue $\times$

5.7 Swap Links - switches

The 'swap' links that connect every game to its six nearest neighbours (See Subsection 2.1 for the logic of similar games.) can be displayed explicitly on the periodic table as curved lines connecting games. There are 432 links so displaying them all at once is too complex to be of use. However, the display of restricted sets of links can be revealing. The main controls are a set of switches for displaying individual swap links.

- "C12?": display the 72 C12 swap links in shades of red.
- "C23?": display the 72 C23 swap links in shades of blue.
- "C34?": display the 72 C34 swap links in shades of green.
- "R12?": display the 72 R12 swap links in shades of red.
- "R23?": display the 72 R23 swap links in shades of blue.

[2]These were summarized in the "Solutions" bar graph that can also be selected in the previous subsection.

- “R34?”: display the 72 R34 swap links in shades of green.

In addition there are two buttons, to the right of the six swap switches, that reset all the switches together.

- “All on”: set all six swap link switches to “On”.
- “All off”: set all six swap link switches to “Off”.

Finally there is one switch for showing only links that connect games that are *adjacent* to each other on the current configuration of the periodic table.

- “Adj-only?”: click switch to “On” to show only swaps of the selected type(s) that connect adjacent games; click switch to “Off’ to show all links of the selected type(s).

The appearance of the links indicates the distance on the table between connected games. Long lines are drawn thinner and in stronger hues than short lines. Note that this distinction is made only to aid in interpreting the links. All links are connecting games that are neighbours.

5.7.1 Some examples of link displays

To demonstrate the use of the links display, here are some examples

1. Reinitialize the display (click “Initialized display”) to restore the standard configuration. Turn all swap links off then turn on “C12?”. Make sure “Adj-only?” is set to “Off” and click “Update display”. 72 thick and pale curves are now shown connecting the games in pairs based on C12 swaps.

   Click “R12?” to “On” and update the display again. Now displayed are the links defining tiles. See Subsection 5.3.

   Now click the “Left” button (See Subsection 4.4) to roll the layers one position. The toroidal shape of the layers is evident as the longer C12 links from edge to edge are shown. Click “Down” to roll the toruses so R12 links also connect opposite edges.

   Click “Adj-only?” to “On” and update the display. The long links connecting opposite edges are no longer shown.

2. Reinitialize the display (click “Initialized display”) to restore the standard configuration. Turn all swap links off then turn on “C12?” and “C23. Make sure “Adj-only?” is set to “Off” and click “Update display”. The display shows the cycle of six games

connected by C12 and C23 swaps. Now click both “R12” and “R23” and redisplay. Now all the links within layers are displayed.

Note the effect of clicking “Adj-only?” on and off.

3. Reinitialize the display (click “Initialized display”) to restore the standard configuration. Click “All On” to turn on all links and make sure “Adj-only” is set of “Off”. When you click to update the display, a complex pattern of links appears with most of the complication caused by the green C34 and R34 links between layers.

   Now click “Adj-only?” to “On” and update the display. Now there are no green links showing because no adjacent games are connected by these 3 and 4 swaps. Click the “Left” button and then the “Down” button. (If you have done this correctly, game 111 should be in the top right corner of layer one at the centre of the display.) There are now green links connecting adjacent games across the layer boundaries. In this configuration, the entire periodic table can be considered as a single torus.

6 Analyzing Experimental Data

To come. In this section, there will be instructions on how to display other data on the periodic table. The Miller data is an example of what is possible.

References

[1] Greenberg, Joseph. *The Theory of Social Situations: an Alternative Game Theoretic Approach*. Cambridge: Cambridge University Press. 1990.

[2] Robinson, D. and Goforth, D. *The Topology of* 2×2 *Games: A New Periodic Table.* Routledge, London. 2005.

[3] Rapoport, Anatol and Melvyn J. Guyer. “A Taxonomy of 2×2 Games” in *General Systems, Vol. XXIII*: 125-136. 1978.

[4] Brams, Steven J. *Theory of Moves.* Cambridge: Cambridge University Press. 1994.

[5] Perlo-Freeman, Samuel. 2006. “The Topology of Conflict and Cooperation” U of the West of England, Dept of Economics, Discussion Paper 0609.