ASSIGNMENT 5, SOLUTIONS

MATH 3416

QUESTION 1(a): PROBLEM 7.1.3(c): Apply naive Gaussian elemination to $\begin{cases} 0x_1 + 2x_2 = 4 \\ x_1 - x_2 = 5 \end{cases}$, account for failures, Solve the system by other means if possible:

ANSWER: Failure becausse of the zero pivot in row 1. To fix this exchange the two rows to get

$$\left.\begin{array}{l} x_1 - x_2 = 5 \\ \quad 2x_2 = 4 \end{array}\right\}$$ The solution is $x_1 = 7$, $x_2 = 2$

QUESTION 1(a): PROBLEM 7.1.3(d): Apply naive Gaussian elemination to $\begin{cases} x_1 + x_2 + 2x_3 = 4 \\ x_1 + x_2 + 0x_3 = 2 \\ 0x_1 + x_2 + x_3 = 0 \end{cases}$ account for failures.

Solve the system by other means if possible

ANSWER:

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & \boxed{0} & -2 & -2 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

We now have a zero pivot so naive gaussian elimination fails: To fix this exchange rows 2 and 3 to get

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -2 & -2 \end{array}\right].$$

Now use back substitution to get $x_3 = 1$, $x_2 = -1$, $x_1 = 4$ so the solution vector is $[4, -1, 1]^T$

QUESTION 1(b): PROBLEM 7.1.4 Solve following system of equations, retaining only 4 significant figures in each step of the calculation, and compare your answer with the solution obtained when eight significant figures are retained. Be consistent by either always rounding to the number of significant figures that are being carried or always chopping.

ANSWER:

$$0.1036\, x_1 + 0.2122\, x_2 = 0.7381$$
$$0.2081\, x_1 + 0.4247\, x_2 = 0.9327$$

Using 4 sig figs and rounding the multiplier is $0.2081/0.1036 = 2.009$. This gives

$$0.1036\, x_1 + 0.2122\, x_2 = 0.7381$$
$$-0.001610\, x_2 = -0.5503$$

This gives the solution $x_1 = -697.3$, $x_2 = 343.9$.

Using 8 sig figs the multiplier is 2.0086873 and the system is

$$0.1036 + 0.2122\, x_2 = 0.7381$$
$$-0.0015434445\, x_2 = -0.54991217$$

and the solution is now $x_1 = -720.79976$, $x_2 = 356.28768$.

These results are significantly different because the system is somewhat ill-conditioned (the determinant of the system is approximately 1.6×10^{-4}).

QUESTION 2(a)

Answer:

$$\begin{bmatrix} 1 & 2 & 4 & 1 \\ 2 & 8 & 6 & 4 \\ 3 & 10 & 8 & 8 \\ 4 & 12 & 10 & 6 \end{bmatrix} \quad \begin{matrix} E_2 \leftarrow E_2 - (2)E_1 \\ E_3 \leftarrow E_3 - (3)E_1 \\ E_4 \leftarrow E_4 - (4)E_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 4 & 1 \\ 0 & 4 & -2 & 2 \\ 0 & 4 & -4 & 5 \\ 0 & 4 & -6 & 2 \end{bmatrix} \quad \begin{matrix} E_3 \leftarrow E_3 - (1)E_2 \\ E_4 \leftarrow E_4 - (1)E_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 4 & 1 \\ 0 & 4 & -2 & 2 \\ 0 & 0 & -2 & 3 \\ 0 & 0 & -4 & 0 \end{bmatrix} \quad E_4 \leftarrow E_4 - (2)E_3$$

This gives the upper triangular matrix $\begin{bmatrix} 1 & 2 & 4 & 1 \\ 0 & 4 & -2 & 2 \\ 0 & 0 & -2 & 3 \\ 0 & 0 & 0 & -6 \end{bmatrix}$ and the LU decomposition is

$$LU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 \\ 4 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 & 1 \\ 0 & 4 & -2 & 2 \\ 0 & 0 & -2 & 3 \\ 0 & 0 & -4 & 0 \end{bmatrix}$$

Solve $Ly = b$ for y using forward substitution to get

$y_1 = 21$, $y_2 = 52 - 2y_1 = 10$, $y_3 = 79 - 3y_1 - y_2 = 6$, $y_4 = 82 - 4y_1 - y_2 - 2y_3 = -24$

This gives the vector $y = [21, 10, 6, -24]^T$

Solve $Ux = y$ for x using backward substitution to get

$x_4 = 4$, $x_3 = -(6 - 3x_4)/2 = 3$, $x_2 = (10 + 2x_3 - 2x_4)/4 = 2$, $x_1 = 21 - 2x_2 - 4x_3 - x_4 = 1$ so the solution vector is $x = [1, 2, \ , 4]^T$

QUESTION 2(b)

Similar to 2(a)

```
% Question3a

A = [ 1,2,3; 4,5,6; 1,0,50 ]

% do the gaussian elimination
% Note that gauss uses a copy of A

[Aupper,r] = gauss(A)

% Solve 3 systems

e1 = [1,0,0]'
e2 = [0,1,0]'
e3 = [0,0,1]'
x1 = gaussSolve(Aupper,r,e1)
x2 = gaussSolve(Aupper,r,e2)
x3 = gaussSolve(Aupper,r,e3)

% check inverse with the matlab inv function

Ainv = [x1,x2,x3]
AinvCheck = inv(A)
```

```
% Question3b

A = [ 1,2,3; 4,5,6; 1,0,50 ]

% do the gaussian elimination
% Note that gauss uses a copy of A

[Aupper,r] = luFactor(A)

% Solve 3 systems

e1 = [1,0,0]'
e2 = [0,1,0]'
e3 = [0,0,1]'
x1 = luSolve(Aupper,r,e1)
x2 = luSolve(Aupper,r,e2)
x3 = luSolve(Aupper,r,e3)

% check inverse with the matlab inv function

Ainv = [x1,x2,x3]
AinvCheck = inv(A)
```

```
>> question3a
A =
     1     2     3
     4     5     6
     1     0    50
Aupper =
    0.2500    0.7500    1.5000
    4.0000    5.0000    6.0000
    0.2500   -1.6667   51.0000
r =
     2     1     3
e1 =
     1
     0
     0
e2 =
     0
     1
     0
e3 =
     0
     0
     1
x1 =
   -1.6340
    1.2680
    0.0327
x2 =
    0.6536
   -0.3072
   -0.0131
x3 =
    0.0196
   -0.0392
    0.0196
Ainv =
   -1.6340    0.6536    0.0196
    1.2680   -0.3072   -0.0392
    0.0327   -0.0131    0.0196
AinvCheck =
   -1.6340    0.6536    0.0196
    1.2680   -0.3072   -0.0392
    0.0327   -0.0131    0.0196
>>
```

```
>> question3b
A =
     1     2     3
     4     5     6
     1     0    50
Aupper =
    0.2500    0.7500    1.5000
    4.0000    5.0000    6.0000
    0.2500   -1.6667   51.0000
r =
     2     1     3
e1 =
     1
     0
     0
e2 =
     0
     1
     0
e3 =
     0
     0
     1
x1 =
   -1.6340
    1.2680
    0.0327
x2 =
    0.6536
   -0.3072
   -0.0131
x3 =
    0.0196
   -0.0392
    0.0196
Ainv =
   -1.6340    0.6536    0.0196
    1.2680   -0.3072   -0.0392
    0.0327   -0.0131    0.0196
AinvCheck =
   -1.6340    0.6536    0.0196
    1.2680   -0.3072   -0.0392
    0.0327   -0.0131    0.0196
>>
```