
Assignment 3, MATH/COSC 3416, Numerical Methods I

Due Date: Friday, Mar. 4, 2011

NOT ALL QUESTIONS WILL BE MARKED

Question 1 (Polynomial interpolation)

- (a) Do Problem 4.1.1, page 146.
- (b) Do this problem again but obtain the Newton form of the interpolating polynomial using Example 3, page 129, as a guide.

Question 2 (Newton's divided difference table)

- (a) Do Problem 4.1.7(a), page 147
- (b) Do Problem 4.1.7(b), page 147

Question 3 (Interpolation errors)

Using the error formulas

$$|f(x) - P_1(x)| \leq \frac{1}{8} \max(f''(x))h^2, \quad \text{linear interpolation}$$

$$|f(x) - P_2(x)| \leq \frac{1}{9\sqrt{3}} \max(f'''(x))h^3, \quad \text{quadratic interpolation}$$

- (a) What is an appropriate size for the interpolation table for the function $\tan x$ on the interval $[0, 1]$ in order that linear interpolation produce an error no larger than 0.5×10^{-6} .
- (b) Answer (a) for quadratic interpolation.

Question 4 (Derivative Formulas)

- (a) Using Taylor series expansions derive the $O(h^2)$ central difference approximation

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

- (b) Using Richardson extrapolation and Taylor series expansions derive the $O(h^4)$ derivative approximation

$$f'(x) = \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$$

Question 5 (Richardson extrapolation)

Consider the Richardson table for derivatives in the form

step size	table			
h	$D(0,0)$			
$h/2$	$D(1,0)$	$D(1,1)$		
$h/2^2$	$D(2,0)$	$D(2,1)$	$D(2,2)$	
$h/2^3$	$D(3,0)$	$D(3,1)$	$D(3,2)$	$D(3,3)$
\vdots	\vdots	\vdots	\vdots	\vdots

where the central difference formula

$$\varphi(h) = \frac{f(x+h) - f(x-h)}{2h}$$

is used to construct the first column using

$$D(n,0) = \varphi(h/2^n)$$

and the following formula

$$D(n,m) = \frac{4^m D(n,m-1) - D(n-1,m-1)}{4^m - 1} \quad \text{(Use for hand calculations)}$$

$$= D(n,m-1) + \frac{D(n,m-1) - D(n-1,m-1)}{4^m - 1} \quad \text{(Use for programming)}$$

is used, for $n \geq m$, to obtain entries in other columns in terms of the entry to their left and the entry above this entry. For example, $D(2,1)$ is obtained in terms of $D(2,0)$ and $D(1,0)$ and $D(3,2)$ is obtained in terms of $D(3,1)$ and $D(2,1)$.

Construct the table for the derivative of $\tan x$ at $x = 0.5$. Choose an initial step size of $h = 1$ and calculate 4 rows by hand using a calculator.