Assignment 2, MATH/COSC 3416, Numerical Methods I
Due Date: Friday, Feb. 4, 2011 in lockerette

Question 1 (The bisection method)

(a) (from a previous exam) By hand (using a calculator) apply 4 iterations of the bisection method for finding a root of the equation $x - \exp(-x) = 0$ using an initial interval $[0.5, 0.6]$.

(b) In the bisection method the number of steps required for a given error tolerance ε can be calculated (see page 82 of text) using

$$\frac{b-a}{2^{n+1}} < \varepsilon$$

where n is the number of steps. Taking logarithms (in any base) gives the smallest value of such an n as

$$n = \left\lfloor \frac{\log(b-a) - \log(2\varepsilon)}{\log 2} \right\rfloor + 1$$

Rewrite our MATLAB `bisect` function in file `bisect.m` on the course home page so that it uses this formula and is called `bisect2` in file `bisect2.m`. The first line of the function will now be

```
function [c] = bisect2(f, x0, x1, tolerance, printflag)
```

NOTE: To do this make a copy of the `bisect.m` file and call it `bisect2.m`. Also change the function name to `bisect2`. Then use the MATLAB editor to modify the file: use the MATLAB `floor` function to calculate `n` and replace the `while` loop by a `for` loop.

(c) (see Computer Problem 3.1.7, page 87) Use the bisection method to determine roots of these functions on the intervals given using `bisect2` (If you didn't get `bisect2` to work properly use `bisect`).

$$\begin{aligned}
f(x) &= x^3 + 3x - 1 && \text{on } [0,1] \text{ using } \varepsilon = 0.5 \times 10^{-6} \\
g(x) &= x^3 - 2\sin x && \text{on } [0.5,2] \text{ using } \varepsilon = 0.5 \times 10^{-5} \\
h(x) &= x + 10 - x\cosh(50/x) && \text{on } [120,130] \text{ using } \varepsilon = 0.5 \times 10^{-4}
\end{aligned}$$

Find each root using the given error tolerance ε.

Question 2 (Newton method)

(a) (from a previous exam) By hand (using a calculator) apply Newton's method with initial guess $x_0 = 0.5$ to find a root of $e^x - 2\cos x = 0$ to at least 5 significant figures.

(b) Use our MATLAB `newton` function (see course home page: `newton.m`) to do part (a).

(c) Do problems 3.2.1 and 3.2.2 on page 101 of textbook.

(d) Do problem 3.2.16 on page 102 of textbook. Do it by hand to obtain at least 3 significant figures.

(e) Do Computer Problem 3.2.5 on page 106 using our `newton` function.

Question 3 (Secant method)

(a) **(See Computer Problem 3.1.11, page 88)** We want to find the first three positive roots of the equation $f(x) = \tan x + \tanh x = 0$.

First create a script file called `question.m` that graphs the functions $y = \tanh x$ and $y = -\tan x$ on the same graph for $0 \leqslant x \leqslant 12$ and $-2 \leqslant y \leqslant 2$. Do this as follows

```
% question3.m
% Find first three positive roots of
% the function f(x) = tanh(x) + tan(x)

% First define data for the plots

x = linspace(0,12,100); % 100 points between 0 and 12
y1 = tanh(x);
y2 = -tan(x);

% plot intersections in window with
% 0 <= x <= 12, and -2 <= y <= 2

plot(x,y1,'-', x,y2,'--');

axis([0,12,-2,2]); % note the square brackets

% Estimate intervals on which the following
% function f changes sign and find first 3 roots using fzero

f = @(x) tanh(x) + tan(x);
format long
```

This graphs $\tanh(x)$ using a solid line and $-\tan(x)$ using a dotted line. The x values of the intersections of these two graphs give the desired roots of $\tanh(x) + \tan(x)$.

Now add 3 lines at the end of your script that use the MATLAB `fzero` function to find the roots by using the graph to find intervals containing the roots on which the function changes sign and use them with `fzero`).

Finally add 3 more lines that use our secant function `secant` (see course home page: `secant.m`) to find the first three positive roots correct to 5 significant figures.

(b) Do problem 3.3.10 on page 120 of textbook.