# Assignment 1, MATH/COSC 3416, Numerical Methods
# Due Date: Friday, January 21, 2011, 11:30 am in lockerette

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NOT ALL QUESTIONS WILL BE MARKED.
MATLAB will be used in this course. It is available in the labs FA358 and FA352 and others as well.

**Question 0 (Matlab Tutorial)**

To familiarize yourself with MATLAB do the introductory tutorial (`tutorial.pdf`) available online at the course web site home page `http://www.cs.laurentian.ca/badams/mc3416` or here.

**Question 1 (the derivative experiment)**

**(a)** Explain the output of the derivative experiment `deriv.m` discussed in class and given on page 10 of the textbook.

**(b)** Try the same experiment for the derivative of $e^x$ at $x = 0.5$ and explain the output. Use a MATLAB script `deriv2.m`.

**(c)** Try the same experiment for the derivative of $\ln(1+x)$ at $x = 0.1$ and explain the output. Use a MATLAB script `deriv3.m`.

**Question 2 (the inverse tangent function)**

**(a)** Establish the validity of the Taylor series

$$\arctan x = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n-1}}{2n-1}, \quad (-1 \le x \le 1)$$

by starting with the series for $1/(1-x)$ (see Page 20), using the integral definition $\int_0^x \frac{dt}{1+t^2}$ of $\arctan x$, and integrating term by term.

**(b)** Recall that $\tan \pi/4 = 1$. Determine the number of terms of the series that need to be summed to ensure that $|4 \arctan 1 - \pi| < 10^{-2}$.

**(c)** Is this series representation useful for the accurate calculation of $\arctan x$? Explain.

**Question 3 (The error function)**

**(a)** Expand the error function

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt,$$

in a series by starting with the series for $e^x$, to obtain the series for $e^{-t^2}$ and integrating term by term.

**(b)** Obtain the Taylor series of $\operatorname{erf}(x)$, accurate to the $x^5$ term, about zero directly using the Taylor series definition

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \cdots.$$

To obtain the first derivative $\operatorname{erf}'(x)$, you will need to use the fundamental theorem of calculus which states that

$$\frac{d}{dx}\int_0^x f(t)\,dt = f(x).$$

**(c)** Evaluate $\operatorname{erf}(1)$ by using terms of the series to order $x^5$. Compare with the value $\operatorname{erf}(1) \approx 0.8427$, which is correct to four decimal places (obtained using the MATLAB erf function).

**Question 4 (Bessel Functions)**

**(a)** The Bessel functions of the first kind are denoted by $J_n(x)$ where $n = 0, 1, 2, \ldots$. An explicit integral formula defining them is

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x\sin\theta - n\theta)\,d\theta$$

Establish the inequality $|J_n(x)| \leqslant 1$ using a property of integrals given by $\left|\int_a^b f(x)\,dx\right| \leqslant \int_a^b |f(x)|\,dx$.

**(b)** It is known that the $J_n(x)$ satisfy the recurrence relation

$$J_{n+1}(x) = \frac{2n}{x}J_n(x) - J_{n-1}(x).$$

which expresses the value of $J_{n+1}(x)$ in terms of the values of the preceding two functions $J_n(x)$ and $J_{n-1}(x)$. Use this equation to write a MATLAB script called called `bessel.m` to compute and display $J_0(1), J_1(1), \ldots, J_{20}(1)$, in a loop, starting from the known accurate values

$$\begin{aligned} J_0(1) &= 0.76519768655797 \\ J_1(1) &= 0.44005058574493 \end{aligned}$$

In the loop you can also display the exact results using the MATLAB function `besselj(n,x)`. You can use `fprintf` (see `deriv.m`) to produce nicely formatted output.

**(c)** Explain why the inequality $|J_n(x)| \leqslant 1$ is violated.

**Question 5 (from Chapter 1)**

Do the following problems from Chapter 1 of the textbook: Problems 1.2, page 32 (problem 13), Computer Problems 1.2, page 38 (problem 14). Note: MATLAB uses `pi` for $\pi$.