Using PPLANE to plot direction fields and solution curves for a system of first order DE’s

PPLANE is a Java applet that can plot the solutions of systems of two first order DE’s of the form

\[
\frac{dx}{dt} = f(t, x, y) \\
\frac{dy}{dt} = g(t, x, y)
\]

where \( x' = \frac{dx}{dt} \) and \( y' = \frac{dy}{dt} \).

**STEP 1**  Go to web site [http://math.rice.edu/~dfield/dfpp.html](http://math.rice.edu/~dfield/dfpp.html) (or just google PPLANE). Click on the PPLANE 2005.10 button to run the applet. Close the copyright text window (click “OK”) and you should have the windows

- PPLANE Equation Window  For entering the two DE’s and the range of variables
- PPLANE Phase Plane  For displaying the direction field and solution curves (orbits)

**STEP 2**  Now enter the data defining the system in the “PPLANE Equation Window”. As an example we will use the predator-prey system

\[
\begin{align*}
x' &= -0.16x + 0.08xy \\
y' &= 4.5y - 0.9xy
\end{align*}
\]

- The left hand side of the system should be set to \( x \) and \( y \) (Note that the prime is automatically shown and should not be entered).
- The right hand side of the first DE can be entered as \(-0.16 \times x + 0.08 \times x \times y\).
- The right hand side of the second DE can be entered as \(4.5 \times y - 0.9 \times x \times y\).
- Set values for the plot size (0 to 10 in both directions)

  - Minimum \( x = 0 \)
  - Maximum \( x = 10 \)
  - Minimum \( y = 0 \)
  - Maximum \( y = 10 \)

Now click the “Graph Phase Plane” button and the direction field appears in the “Phase Plane Window”. It can be printed by choosing “Print” from the “PPLANE phase plane” File menu.
STEP 3  To plot the single solution satisfying $x(0) = 4$ and $y(0) = 4$:

- From the “Solution” menu select “Keyboard input of initial value”
- From the window that appears use 4 for $x$ and 4 for $y$ (don’t close this window using X).
- Click the “Solve” button and the solution curve appears on the direction field. It should be a closed oval shaped curve that indicates that the solution is periodic.
- Another way to select an initial value is to simply click somewhere in the phase plane. Try this for several values. Each time you will get a closed curve that indicates a periodic solution.

STEP 4  The phase plane shows $x$ and $y$. To see $x$ as a function of $t$:

- From the “Options” menu select “Show cross-hairs”. The cross-hairs will now move with the mouse.
- From the “Graph” menu select “$x$ vs $t$”.
- You will be prompted to click on one of the oval curves.
- Click at the point $x = 4$ and $y = 4$.
- In a separate window you will see a blue curve showing $x$ as a function of $t$.
- When you want to close this window do not click the X. Instead click the “Go away” button (not yet though).
- On this “Graph window” there are options to plot “$y$ vs $t$” and “both”, The curve representing $y$ as a function of $t$ will be red. Try these options.
- Note that both the blue and the red curves are periodic with the same period but shifted.
- As you move the mouse in the “Graph window” a vertical line moves with it. This can be used to crop the graph. Click the vertical line at two values of $t$ and then click on the “crop” button.

**WARNING**  To exit PPLANE use the “Quit” item on the “File” menu. If you click the “X” to close the window your browser may take a while to close the windows.