Classical construction of Sierpinski gasket

The Sierpinski gasket is the classical fractal obtained from an equilateral triangle as indicated in Figure 1.

![Classical construction of the Sierpinski gasket. Start with a black equilateral triangle and successively remove the inner triangle. The remaining black part is the Sierpinski gasket.](image)

Figure 1: Classical construction of the Sierpinski gasket. Start with a black equilateral triangle and successively remove the inner triangle. The remaining black part is the Sierpinski gasket.

We begin with the black triangle shown in Figure 1(a). The first step is to divide the triangle into four identical equilateral triangles, by bisecting the sides of the original triangle, and remove the middle triangle, as shown in Figure 1(b). This process is repeated on each of the resulting black triangles as shown in Figure 1(c). The process is repeated indefinitely and the resulting black figure is called the Sierpinski gasket.

Chaos game construction of Sierpinski gasket

To play the Chaos game, start with the triangle shown in Figure 2 with vertices labeled 0, 1, and 2, with coordinates \((X_0,Y_0), (X_1,Y_1),\) and \((X_2,Y_2)\), respectively. Pick any point \(q\) inside the triangle as the initial current point. Now generate a random number from the set \(\{0,1,2\}\) representing the three vertices of the triangle. If 1 was chosen then locate a new current point \(q\) at the midpoint of the line joining \(p\) to vertex 1. Continue in this fashion, plotting all the points generated.

It is not at all obvious that this iteration process generates the Sierpinski gasket. A first guess might be that the entire triangle is filled with points.
Figure 2: Initial configuration for the chaos game. An arbitrary point $p$ inside the triangle is chosen as initial current point and plotted. Next a random vertex number from $\{0,1,2\}$ is generated and the midpoint $q$ of the line from this vertex to $p$ is plotted and made the next current point. This procedure is repeated indefinitely.

**Algorithm for the Chaos Game**

In the following algorithm the initial current point $(CP_x, CP_y)$ is defined by the point

$$CP_x = \frac{X_0 + X_1}{2}; \quad CP_y = \frac{Y_0 + Y_1}{2}$$

which is inside the triangle.

We assume that a routine called PlotPoint is available such that PlotPoint$(x,y)$ plots the point at position $(x,y)$, where the origin is at the lower left corner.

We also assume that the routine called Random generates random numbers from the set $\{0, 1, 2\}$.

Draw the initial triangle

$CP_x \leftarrow (X_0 + X_1)/2; \; CP_y \leftarrow (Y_0 + Y_1)/2$

PlotPoint($CP_x, CP_y$)

for $k \leftarrow 1$ to maxPoints do

$\quad r \leftarrow$ Random$(0,2)$

$\quad CP_x \leftarrow (CP_x + X_r)/2; \; CP_y \leftarrow (CP_y + Y_r)/2$

PlotPoint($CP_x, CP_y$)

endfor

The results of the chaos game for 1000 points is given in Figure 3 and the results for 40,000 points is given in Figure 4.
Figure 3: Results of chaos game after 1000 points.

Figure 4: Results of chaos game after 40,000 points.