1. (a) (4 marks) Find the general solution of the linear DE \( x \frac{dy}{dx} - 4y = x^6 e^x \).

Answer:

(b) (1 mark) Find the particular solution satisfying the initial condition \( y(1) = 1 \).

Answer:
2. (a) (4 marks) Find the particular solution of $\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y - 1)}$ satisfying the initial condition $y(0) = -1$ and expressed in the explicit form $y = \phi(x)$.

Answer:

(b) (1 mark) Determine the interval on which the solution exists.

Answer:
3. (a) (1 mark) Show that \((6xy - y^3)dx + (4y + 3x^2 - 3xy^2)dy = 0\) is exact.
   Answer:

   (b) (4 marks) Find the general solution.
   Answer:
4. (a) \(3\) mark) Find the particular solution of \(2y'' - 5y' - 3y = 0\) satisfying the initial conditions \(y(0) = 1\) and \(y'(0) = 2\).
   Answer:

   (b) \(1\) mark) Find the general solution of \(y'' - 10y' + 25y = 0\).
   Answer:

   (c) \(2\) marks) Find the general solution of \(y'' + 4y' + 7y = 0\).
   Answer:
5. (6 marks) Solve $y'' + 2y' + 5y = 3 \sin 2t$ using the method of undetermined coefficients.

Answer:
6. **(6 marks)** Solve $y'' + \frac{1}{4}y = \frac{1}{2} \sec(t/2)$ using the variation of parameters method.

   **Answer:**
7. (a) (2 marks) Evaluate the Laplace transform of \( f(t) = \begin{cases} 0, & 0 \leq t < 4 \\ 3, & t \geq 4 \end{cases} \) using the definition of the Laplace transform.

Answer:

(b) (2 marks) Evaluate \( \mathcal{L}^{-1} \left\{ \frac{3s + 6}{s^2 + 4} \right\} \).

Answer:

(c) (1 mark) Evaluate \( \mathcal{L} \{ e^{-3t} \cos 2t \} \).

Answer:
8. (6 marks) Use the Laplace transform to solve the differential equation
\[ y'' - 3y' + 2y = e^{-4t}, \quad y(0) = 1, \quad y'(0) = 5 \]

Answer:
9. (a) (4 marks) Write the following system in matrix form and solve it using eigenvalues and eigenvectors.

\[
\begin{align*}
\frac{dx}{dt} &= 2x + 3y \\
\frac{dy}{dt} &= 2x + y
\end{align*}
\]

Answer:

(b) (2 marks) Sketch the phase portrait of the system.
Some Integrals

\[
\int u e^u \, du = (u - 1)e^u + C \\
\int \tan u \, du = \ln |\sec u| + C
\]

Some Laplace Transforms

<table>
<thead>
<tr>
<th>(\mathcal{L}{1} = \frac{1}{s})</th>
<th>(\mathcal{L}{t^n} = \frac{n!}{s^{n+1}})</th>
<th>(\mathcal{L}{e^{at}} = \frac{1}{s-a})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mathcal{L}{\sin kt} = \frac{k}{s^2 + k^2})</td>
<td>(\mathcal{L}{\cos kt} = \frac{s}{s^2 + k^2})</td>
<td></td>
</tr>
</tbody>
</table>

Laplace Transforms of Derivatives

\[
\mathcal{L}\{f'(t)\} = sF(s) - f(0) \\
\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0) \\
\ldots \\
\mathcal{L}\{f^{(n)}(t)\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)
\]

Translation of a Laplace Transform

\[
\mathcal{L}\{e^{at}f(t)\} = F(s-a) \quad \text{or} \quad \mathcal{L}\{e^{at}f(t)\} = \mathcal{L}\{f(t)\}|_{s \to s-a} \\
\mathcal{L}^{-1}\{F(s-a)\} = \mathcal{L}^{-1}\{F(s)|_{s \to s-a}\} = e^{at}f(t)
\]