MATH 2066 EL-01 – DIFFERENTIAL EQUATIONS – FINAL EXAM

Thursday, December 17, 2009, 14:00 (2:00 pm)

Time Allowed: 3 hours *Instructor:* Barry G. Adams

Name (PLEASE PRINT) ____

Student # ____

1. *Any calculator permitted.*
2. *No other aids permitted.*
3. *Exam booklets are NOT required.*
4. *Number of questions: 9*
5. *Total Marks: 50*

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1. (a) **(4 marks)** Find the general solution of the linear DE $x\dfrac{dy}{dx} - 4y = x^6 e^x$.

   Answer:

   (b) **(1 mark)** Find the particular solution satisfying the initial condition $y(1) = 1$.

   Answer:

2. (a) **(4 marks)** Find the particular solution of $\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}$ satisfying the initial condition $y(0) = -1$ and expressed in the explicit form $y = \phi(x)$.
Answer:

(b) **(1 mark)** Determine the interval on which the solution exists.
Answer:

3. (a) **(1 mark)** Show that $(6xy - y^3)dx + (4y + 3x^2 - 3xy^2)dy = 0$ is exact.
   Answer:

   (b) **(4 marks)** Find the general solution.
   Answer:

4. (a) **(3 mark)** Find the particular solution of $2y'' - 5y' - 3y = 0$ satisfying the initial conditions $y(0) = 1$ and $y'(0) = 2$.

   Answer:

   (b) **(1 mark)** Find the general solution of $y'' - 10y' + 25y = 0$.

   Answer:

   (c) **(2 marks)** Find the general solution of $y'' + 4y' + 7y = 0$.

   Answer:

5. **(6 marks)** Solve $y'' + 2y' + 5y = 3\sin 2t$ using the method of undetermined coefficients.

   Answer:

6. **(6 marks)** Solve $y'' + \frac{1}{4}y = \frac{1}{2}\sec(t/2)$ using the variation of parameters method.

   Answer:

7. (a) **(2 marks)** Evaluate the Laplace transform of $f(t) = \begin{cases} 0, & 0 \le t < 4 \\ 3, & t \ge 4 \end{cases}$ using the definition of the Laplace transform.

   Answer:

   (b) **(2 marks)** Evaluate $\mathscr{L}^{-1}\left\{\dfrac{3s+6}{s^2+4}\right\}$.

   Answer:

   (c) **(1 mark)** Evaluate $\mathscr{L}\{e^{-3t}\cos 2t\}$

   Answer:

8. **(6 marks)** Use the Laplace transform to solve the differential equation
$$y'' - 3y' + 2y = e^{-4t},\ y(0) = 1,\ y'(0) = 5$$

Answer:

9. (a) **(4 marks)** Write the following system in matrix form and solve it using eigenvalues and eigenvectors.

$$\begin{aligned} \frac{dx}{dt} &= 2x + 3y \\ \frac{dy}{dt} &= 2x + y \end{aligned}$$

Answer:

(b) **(2 marks)** Sketch the phase portrait of the system.

Some Integrals

$\int ue^u \, du = (u-1)e^u + C$	$\int \tan u \, du = \ln \lvert \sec u \rvert + C$

Some Laplace Transforms

$\mathscr{L}\{1\} = \frac{1}{s}$	$\mathscr{L}\{t^n\} = \frac{n!}{s^{n+1}}$	$\mathscr{L}\{e^{at}\} = \frac{1}{s-a}$
$\mathscr{L}\{\sin kt\} = \frac{k}{s^2+k^2}$	$\mathscr{L}\{\cos kt\} = \frac{s}{s^2+k^2}$	

Laplace Transforms of Derivatives

$$\begin{aligned} \mathscr{L}\{f'(t)\} &= sF(s) - f(0) \\ \mathscr{L}\{f''(t)\} &= s^2F(s) - sf(0) - f'(0) \\ &\cdots \\ \mathscr{L}\{f^{(n)}(t)\} &= s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0) \end{aligned}$$

Translation of a Laplace Transform

$$\mathscr{L}\{e^{at}f(t)\} = F(s-a) \qquad \text{or} \qquad \mathscr{L}\{e^{at}f(t)\} = \mathscr{L}\{f(t)\}|_{s \to s-a}$$

$$\mathscr{L}^{-1}\{F(s-a)\} = \mathscr{L}^{-1}\{F(s)|_{s \to s-a}\} = e^{at}f(t)$$